Solving the 3D Stokes System on a Variable Resolution Mesh

Todd Ringler
Theoretical Divison

Climate, Ocean, and Sea Ice Modeling Project
http://public.lanl.gov/ringler/ringler.html

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3. The majority of this effort is being undertaken in the university setting, so student training is a significant aspect of the project.

The project setting ...

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- Stokes flow
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Interested Members of the Ice Sheet Community.

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3) Aligns nicely with LANL COSIM mission space.

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3) We think we can formulate the system so that we can recover reduced-order systems if we needed.

What is our process?

Two tracks: model develop and mesh generation

3. Iterate

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5. Construct full-up model

Model Requirements

Ice Sheet Modeling:
Governing Equations,
Requirements and Methods

We have a working document …
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1. What equations do we want to solve?
2. Physical system requirements
3. Computational system requirements
4. Modeling system requirements
5. Proposed interfaces

Model Requirements: Physical Requirements

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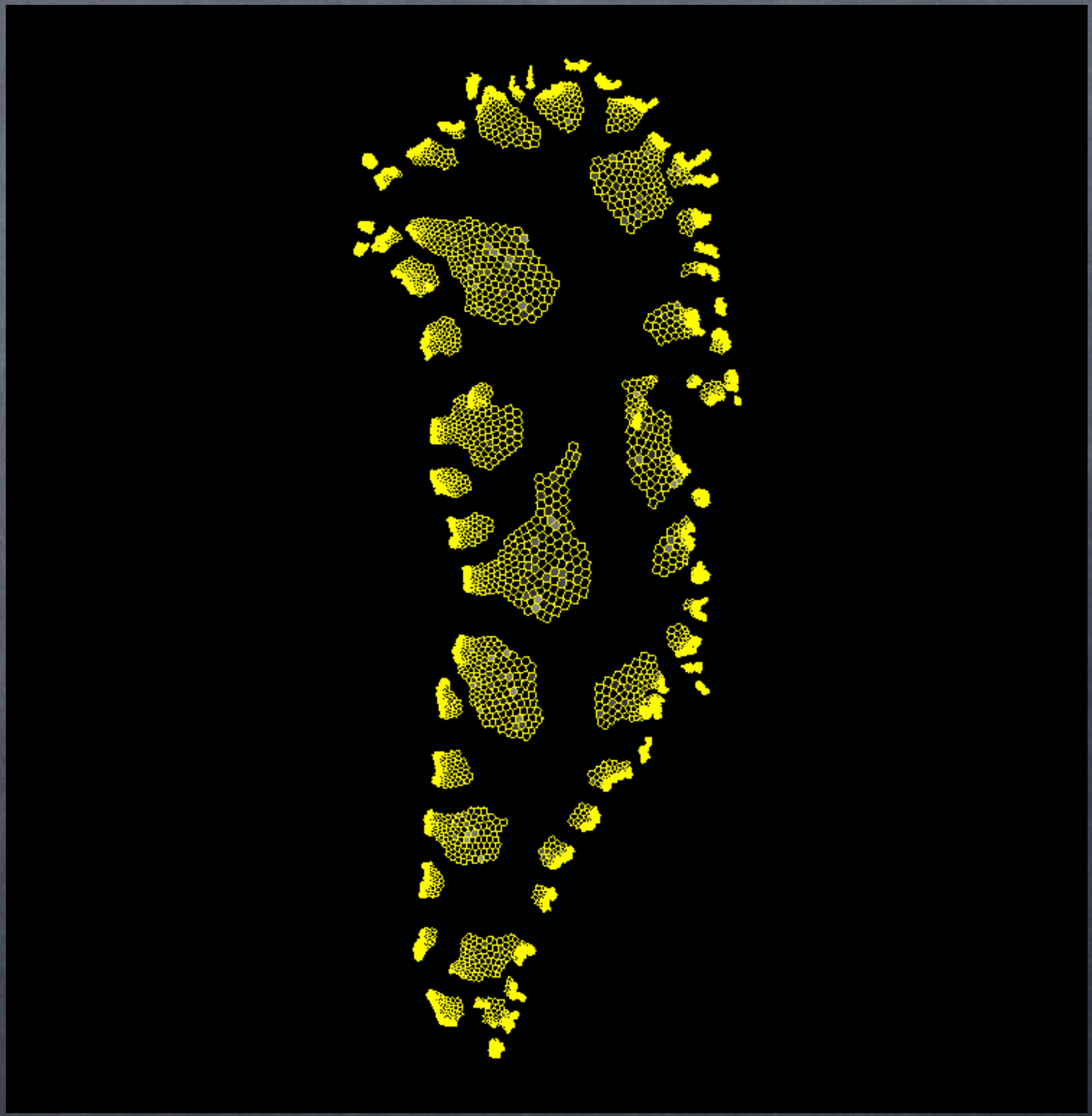
Ability to faithfully simulate regions of rapid motion and regions of rapid transition.

Ability to recover reduced-order systems.

Model Requirements: Computational Requirements

For the combined Greenland/Antarctic systems, a throughput of 100 simulated years per wall clock day is required. Assume access to 1000 dedicated processors.

For example, break the domain into "blocks" and send each block to its own cpu.



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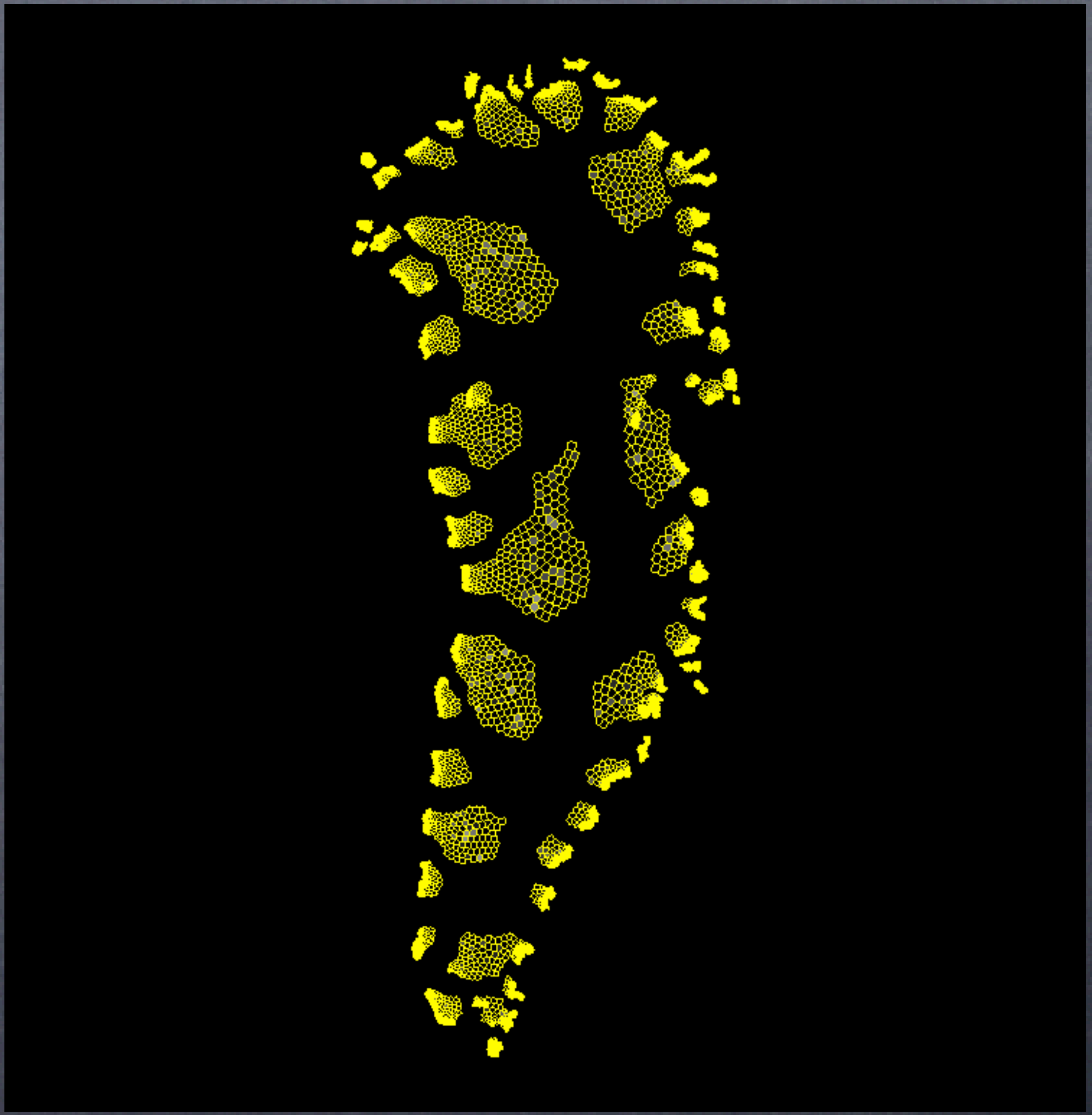

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(The LANL ocean model (POP) can use 10,000s of cpu with this method.)

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FORTRAN95, MPI and OpenMP.

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We are still unclear as to the best choice for our vertical coordinate for the Stoke system (stacked or 3D unstructured). Is a fully unstructured formulation such a radical notion?

Where does "Model Develop" effort stand?

1) We have a draft requirements document.

2) We have identified a plausible method.

3) We are conducting analysis of the method
to better insure it meets our requirements.

On to mesh generation and some results ....

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2) These disparate scales of motion are relatively stationary on the time-scale of decades.

Mesh Generation

Can we develop a meshing technique with the following properties?

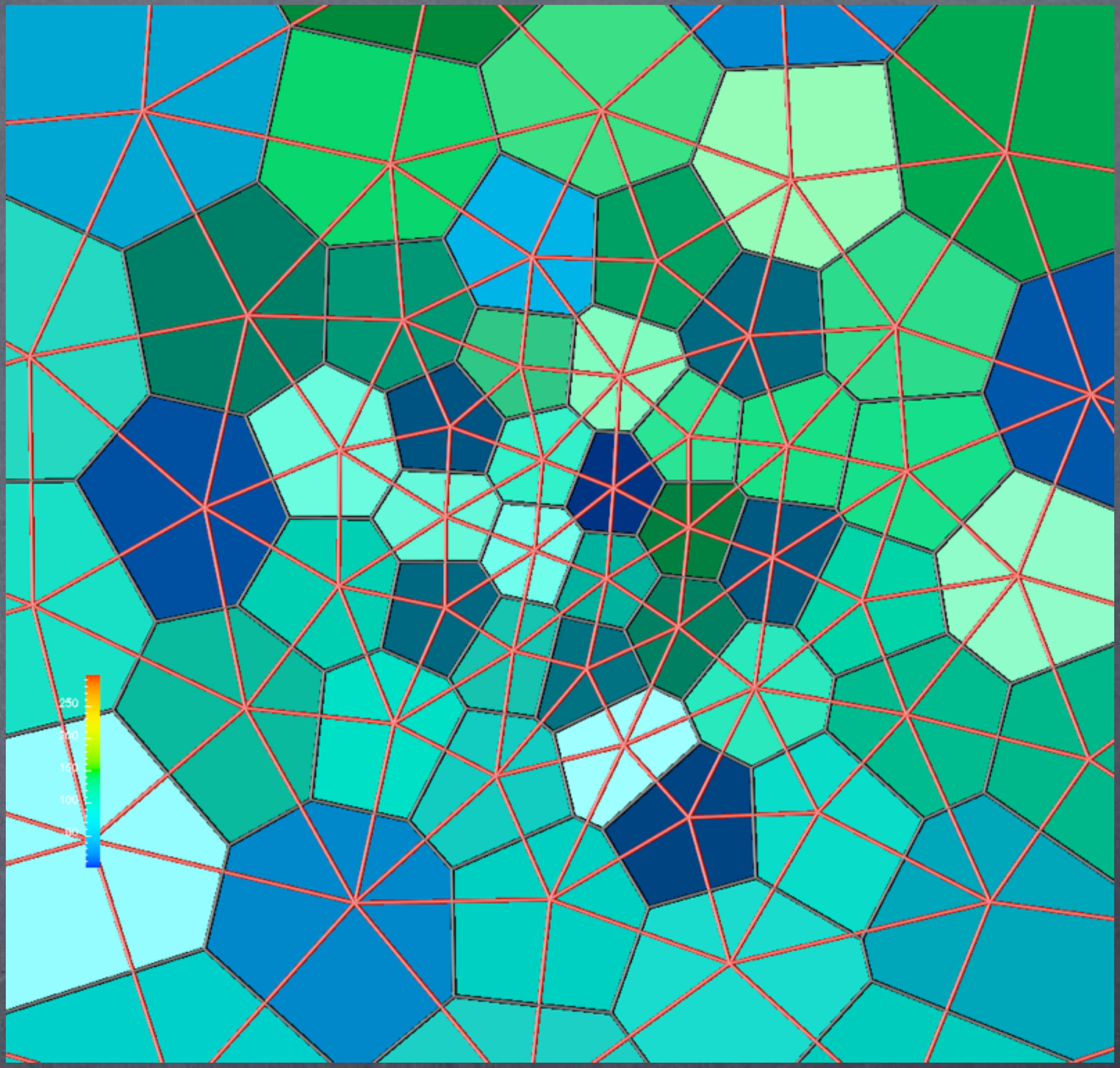
1) Puts degrees of freedom where they are most needed.

2) Has some guarantees related to mesh quality.

3) Is accessible to the ice sheet modeling community.

4) Leads to a better simulation.

Spherical Centroidal Voronoi Tessellations (SCVT) have the potential to meet these requirments.



Voronoi Diagram (colored)
and the dual Delaunay triangulation

For more information ....

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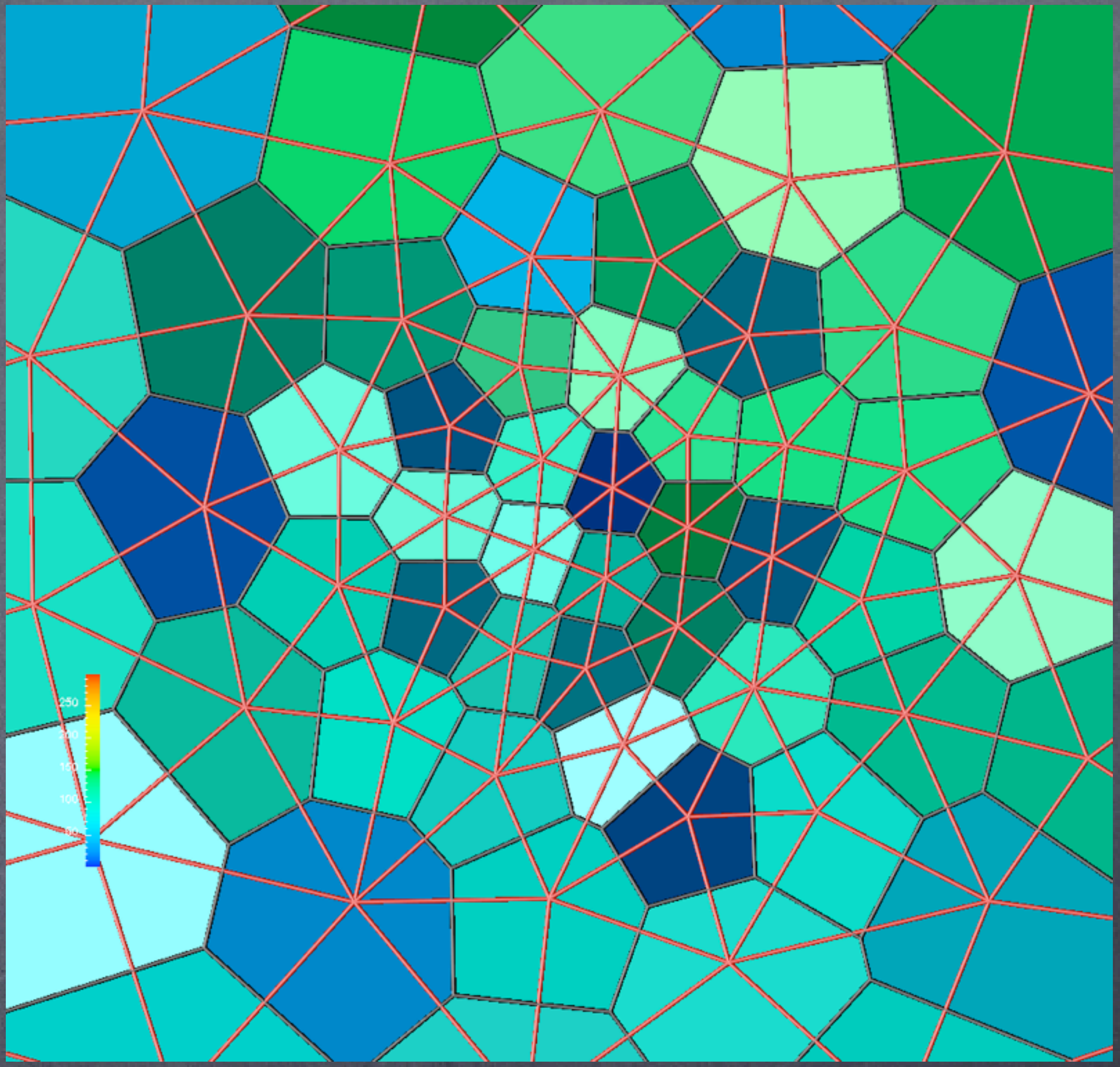
SCVTs and their close relatives have already been successfully used in climate system modeling.

To date, the primary motivation for their use has been the mesh uniformity when tiling the sphere.

I think their potential goes well beyond the ability to produce a globally uniform mesh.



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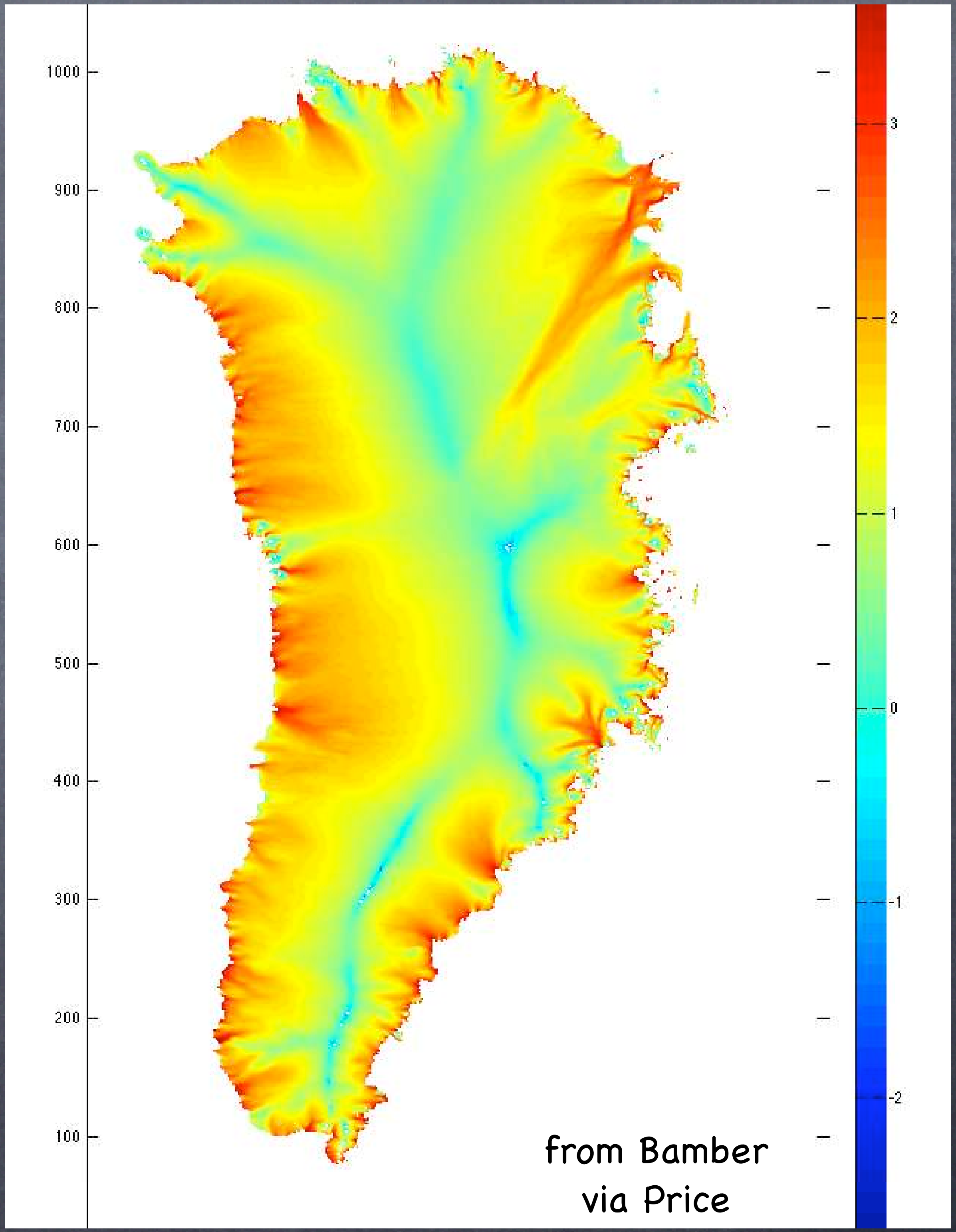
SCVT by example ....

log10 (sfc velocity)



SCVT by example ....

Much of the ice sheet is quiescent.

The real action is in and around the ice stream zones that are clearly defined in the data set.

Can we create a high-quality mesh with ~1 km resolution in the ice stream zones and ~50 km resolution in quiescent regions?

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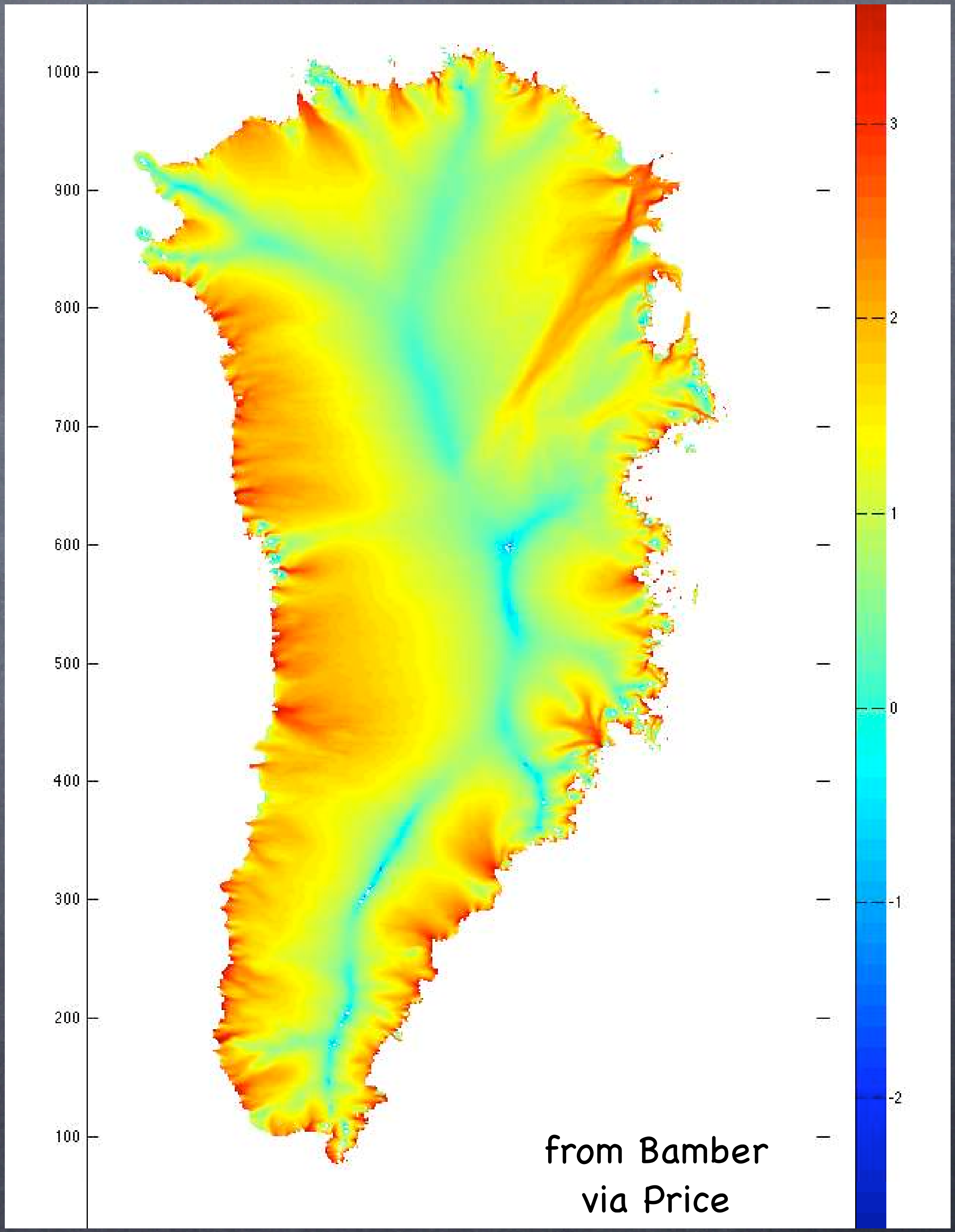

SCVT .... how it works

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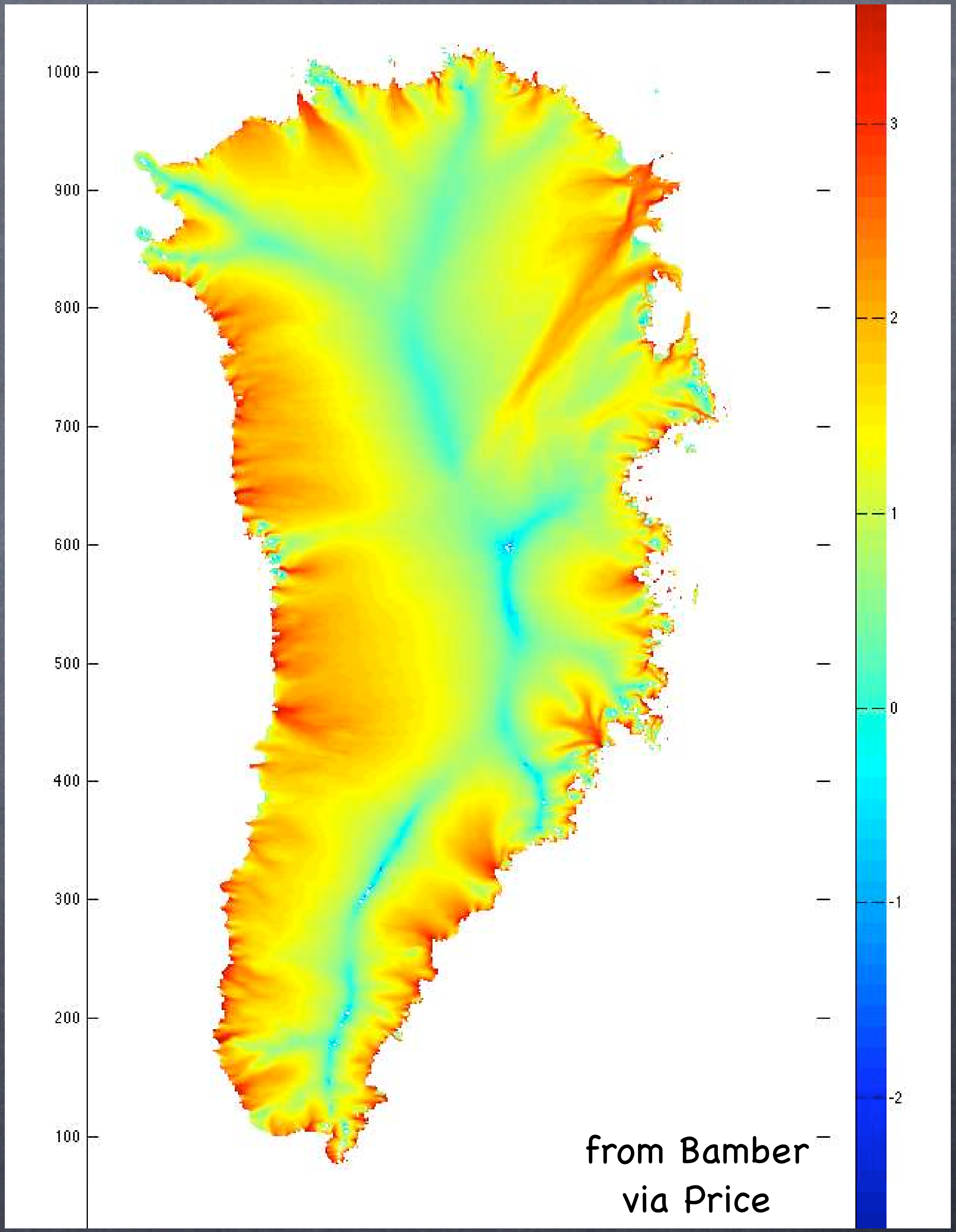

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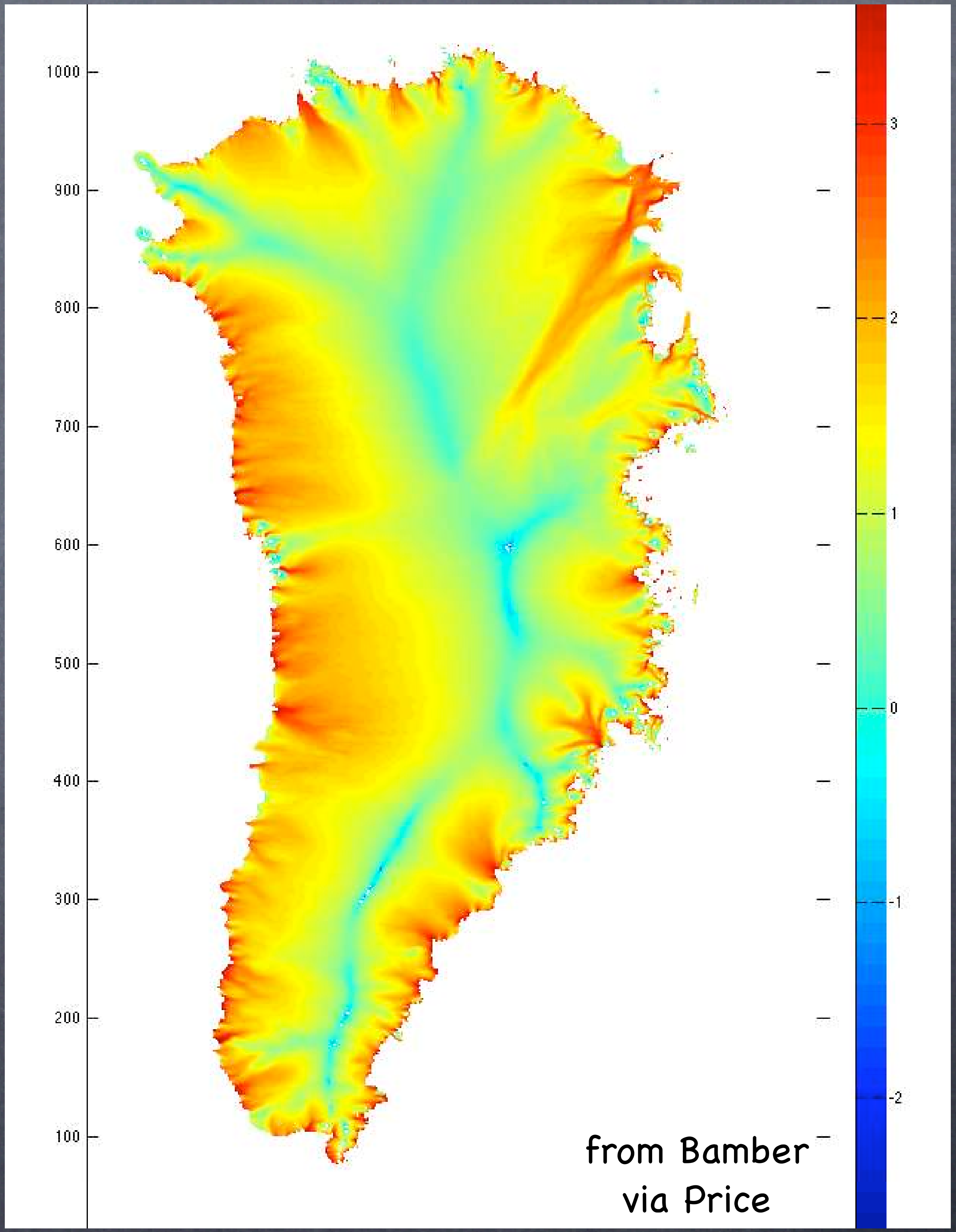

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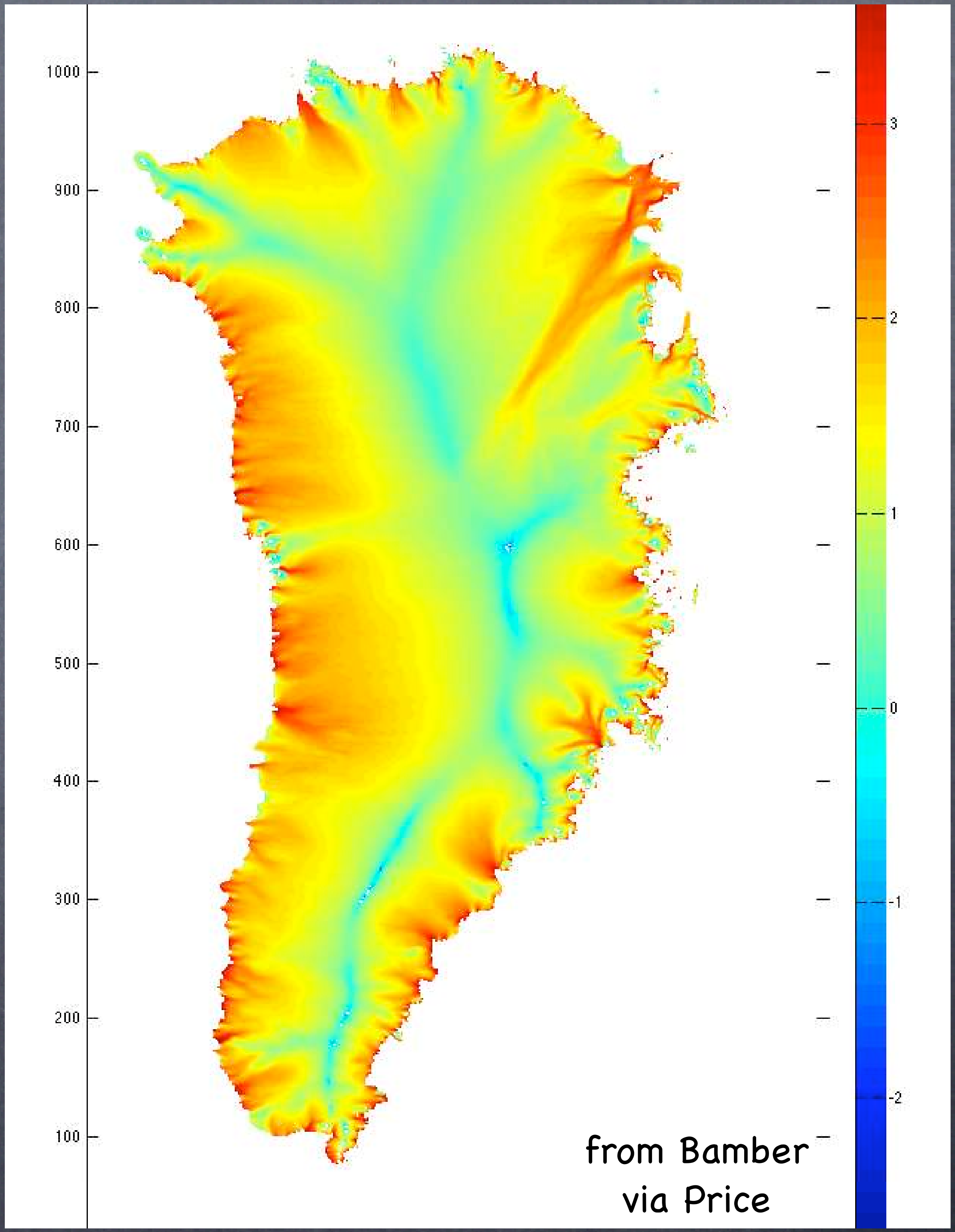

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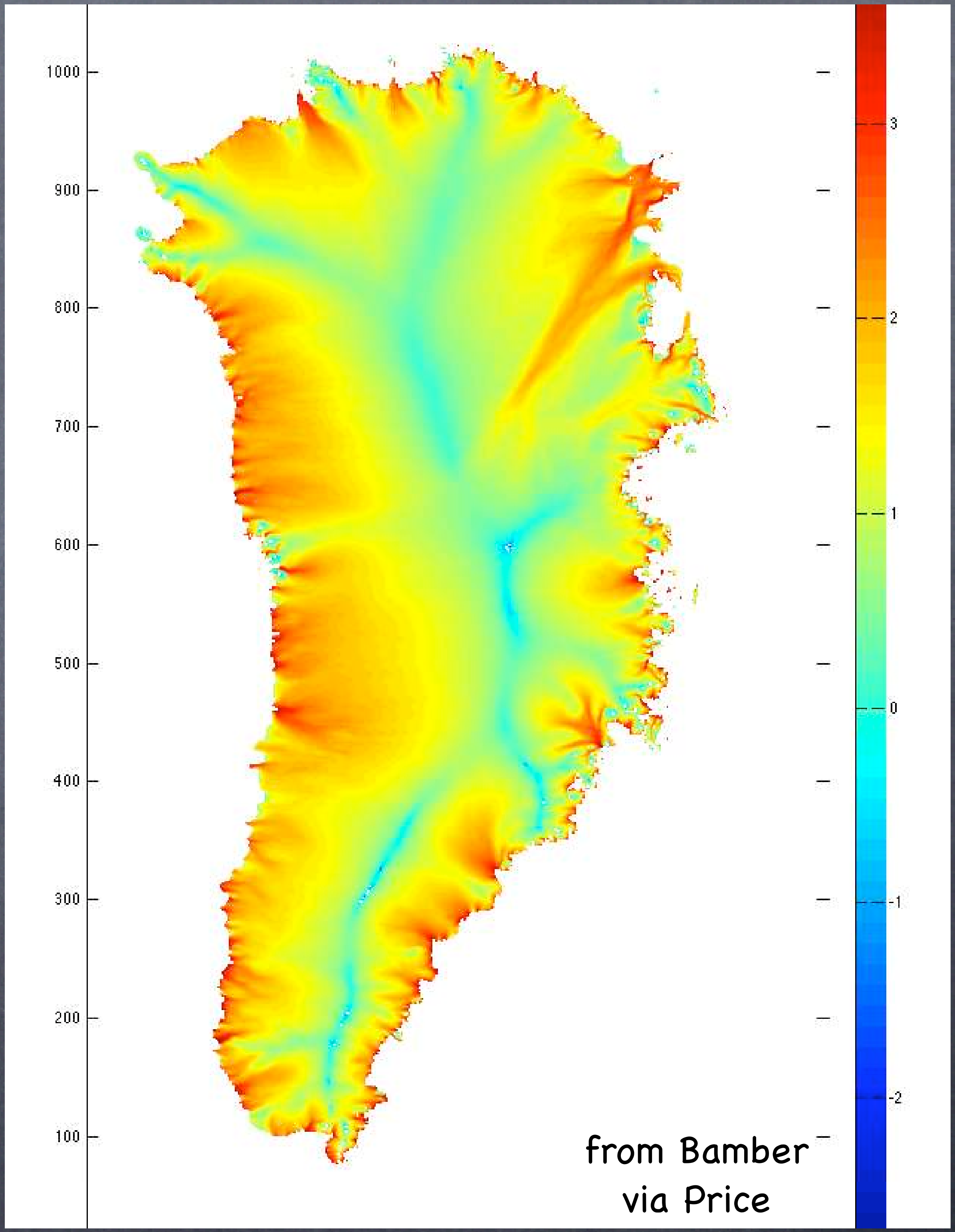

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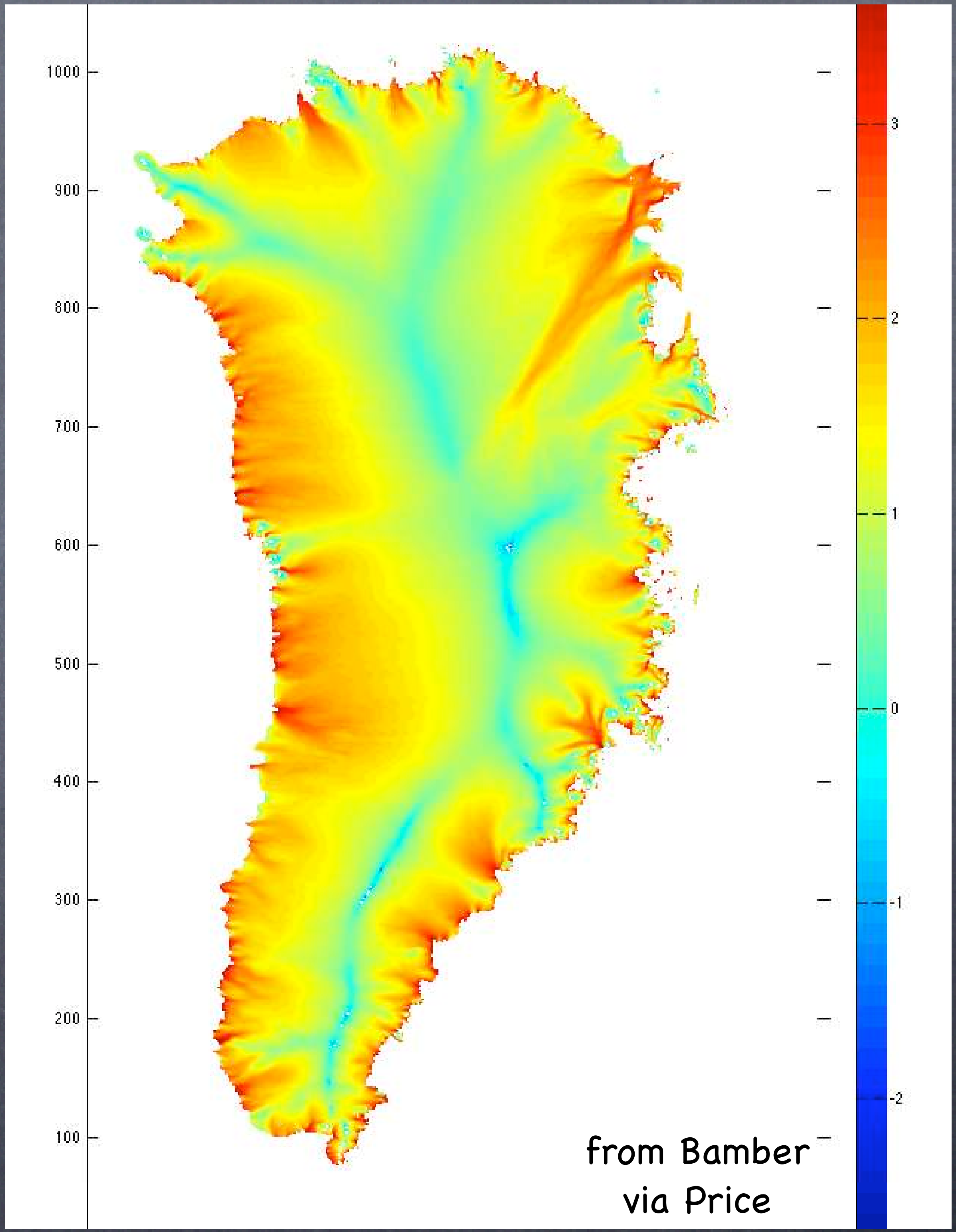

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Build mesh. (start-to-finish is ~hours).

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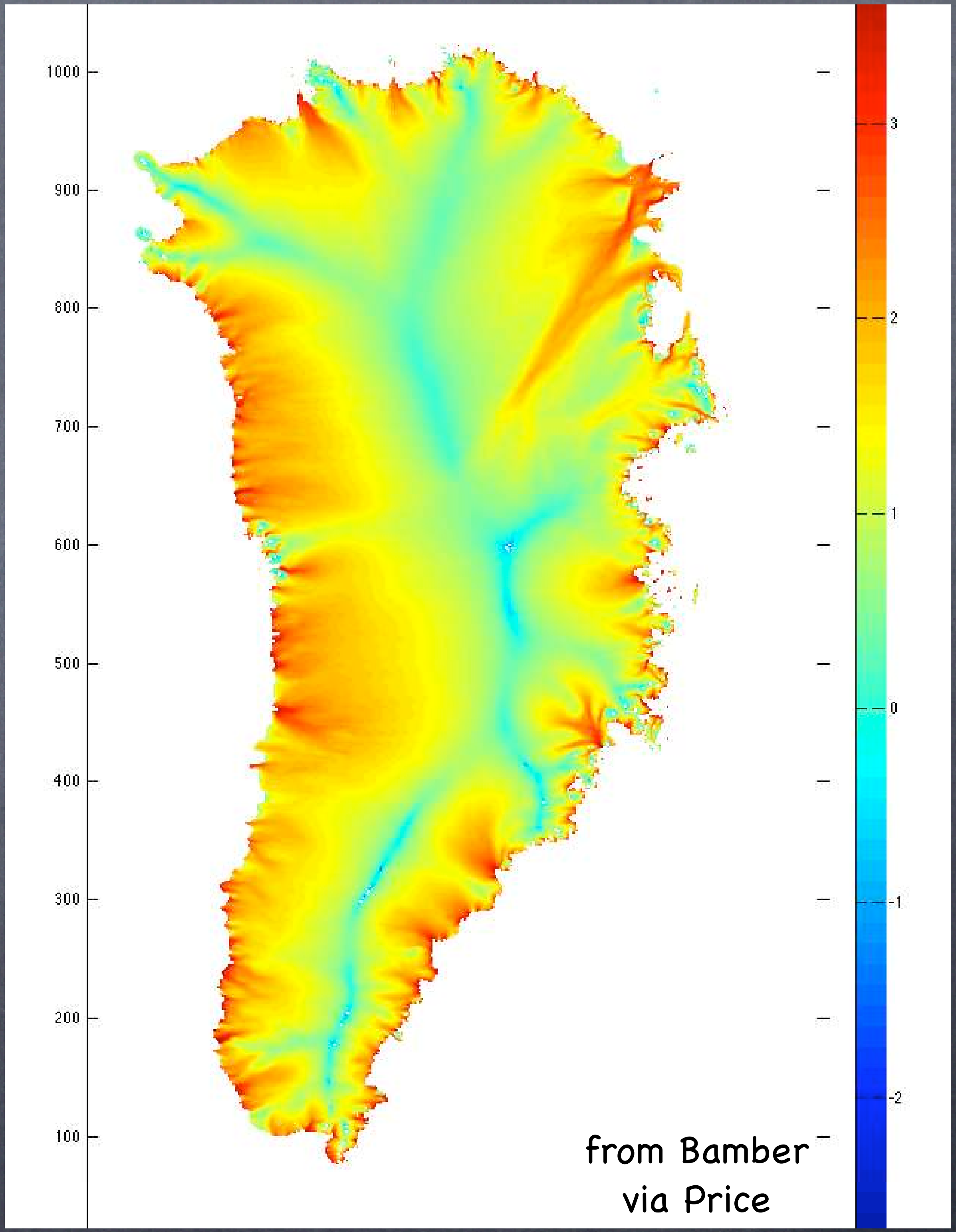
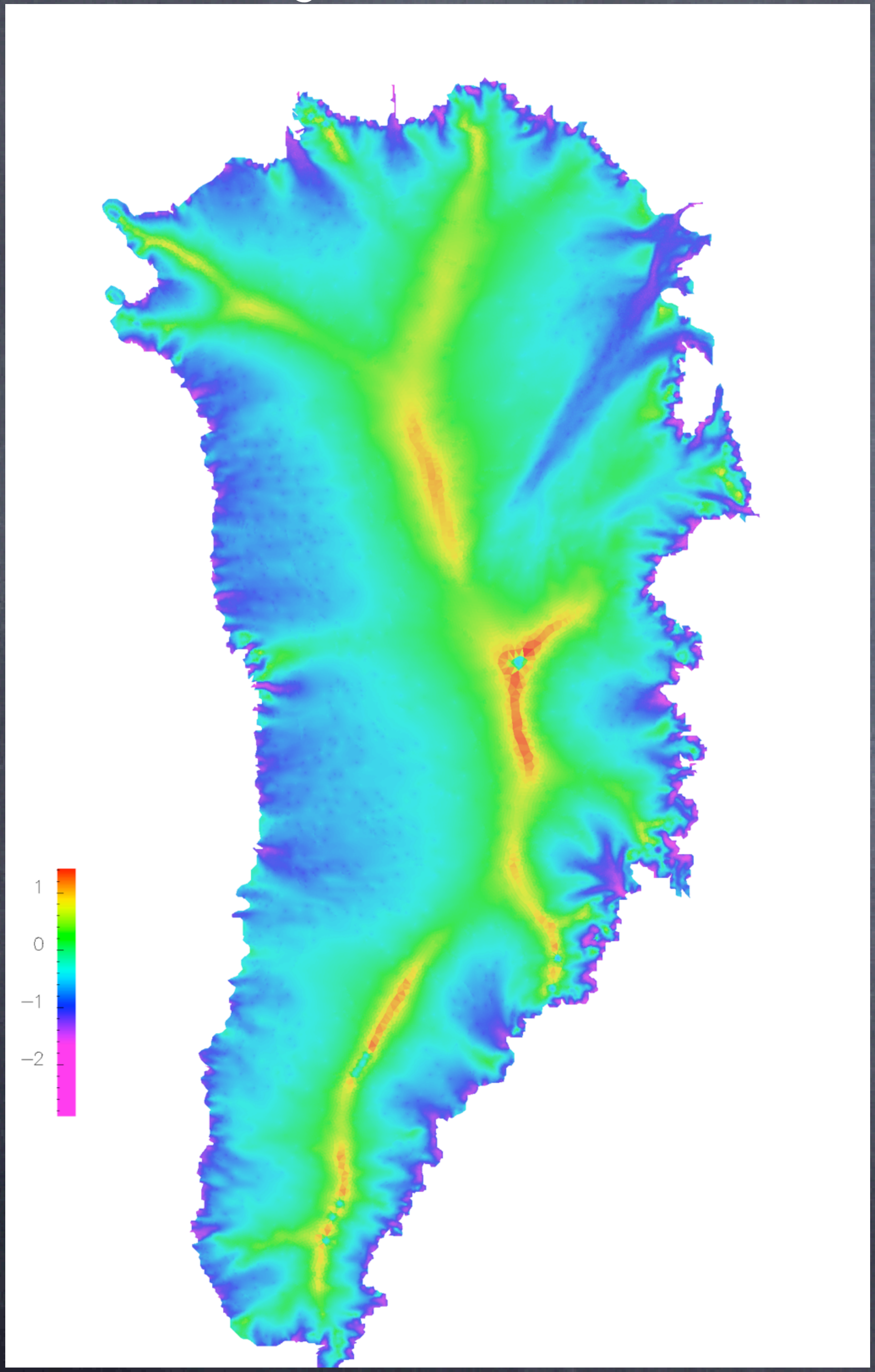


Results ....

log10 (cell area)



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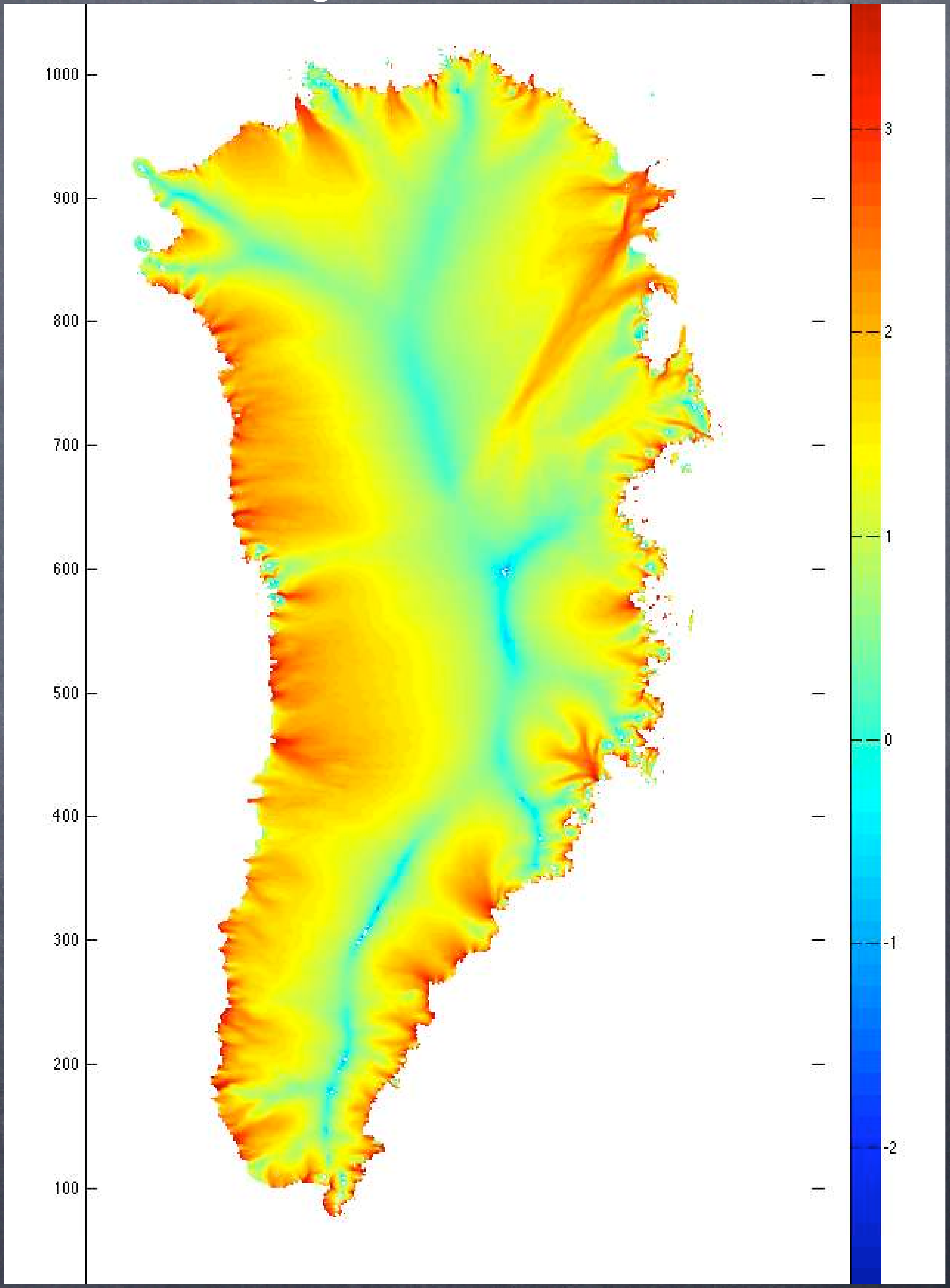

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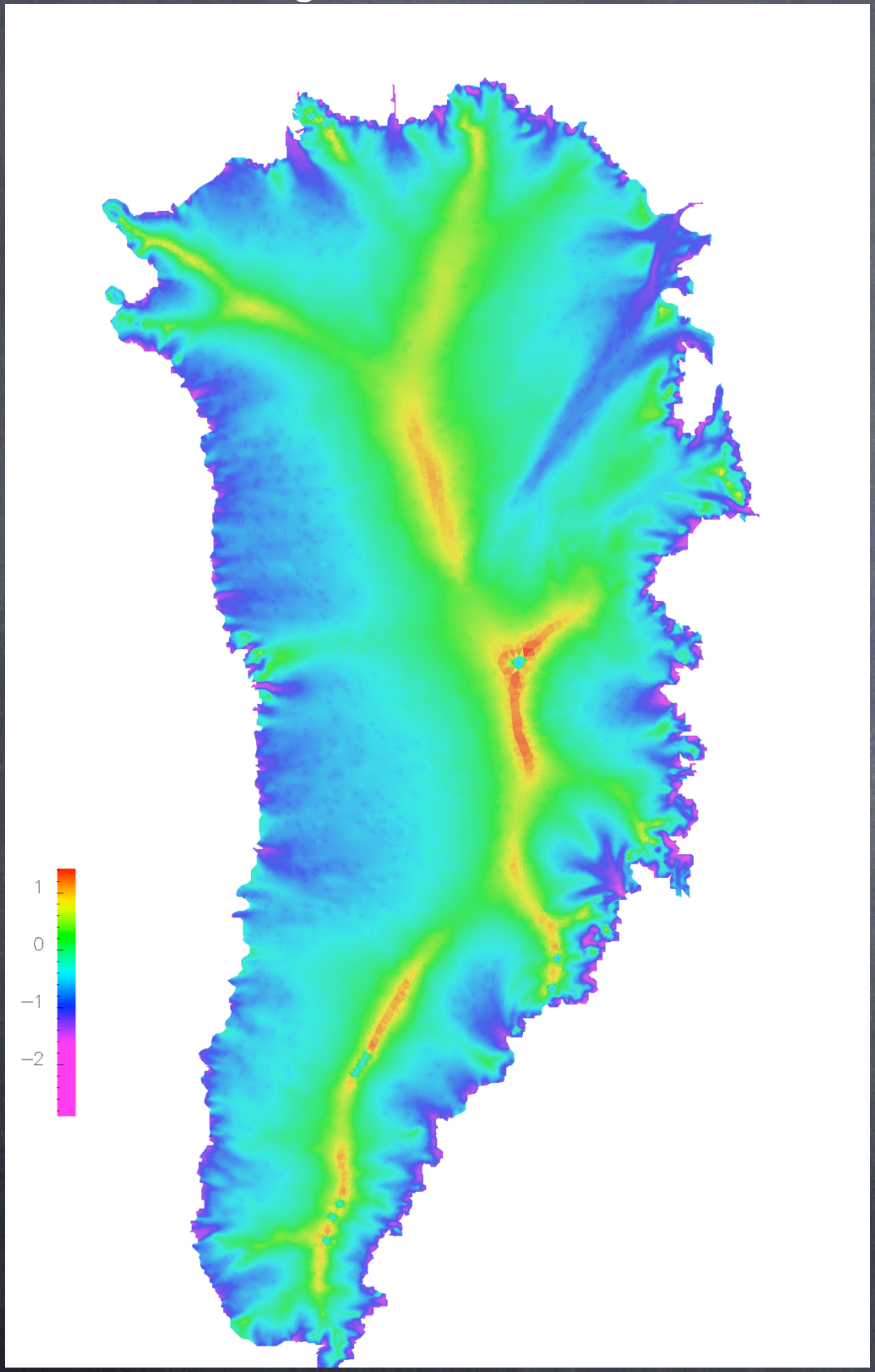

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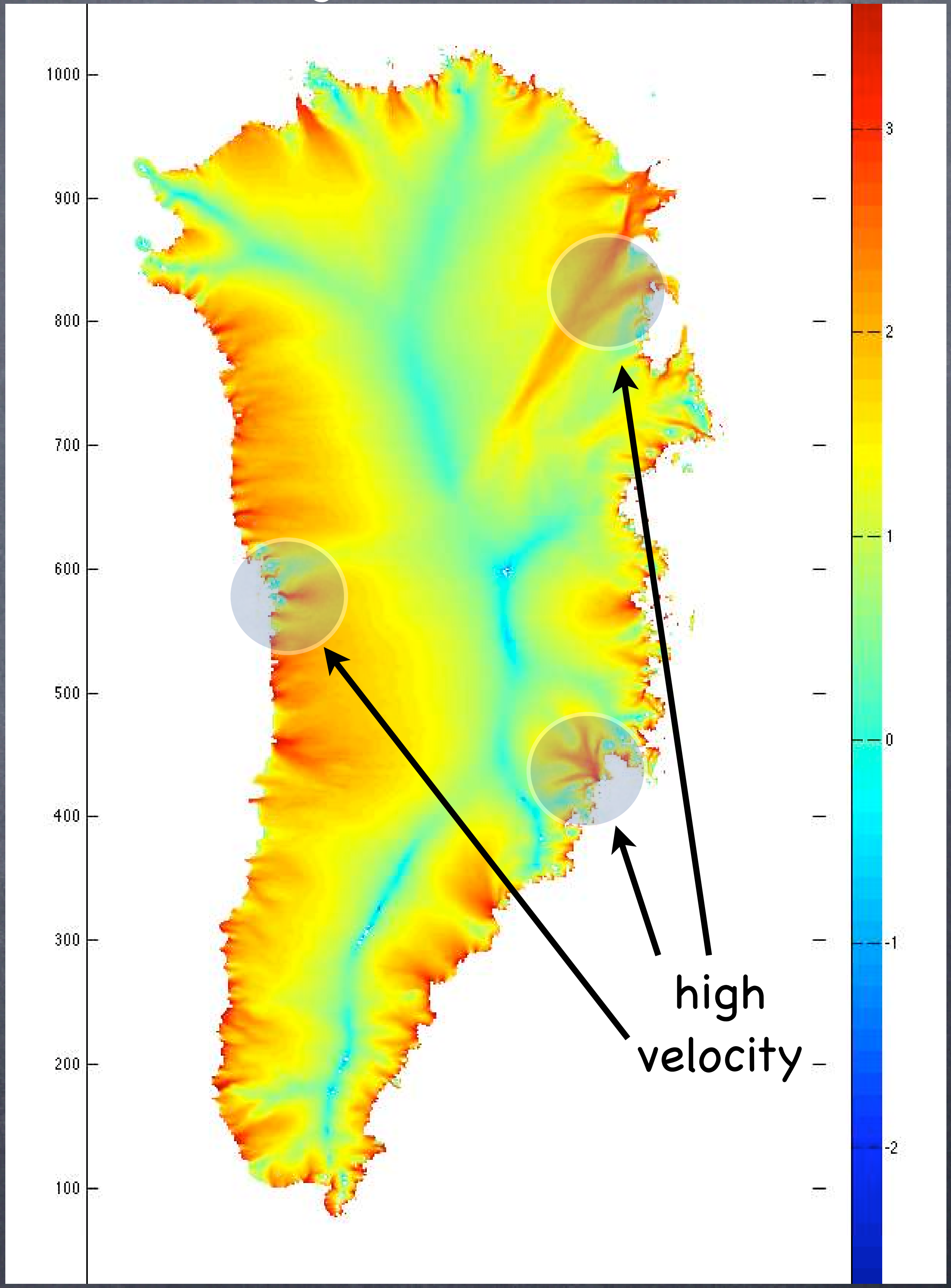

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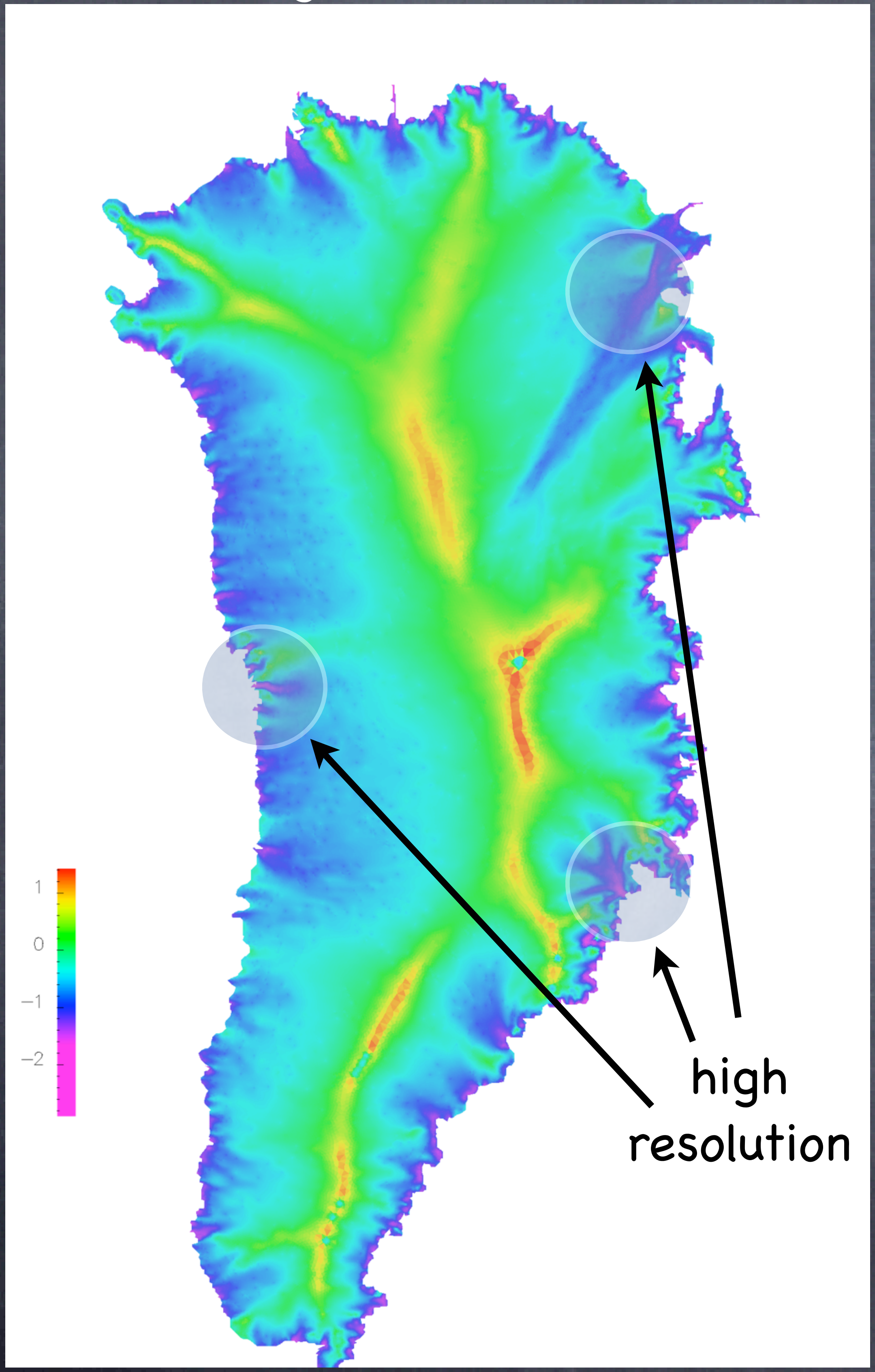

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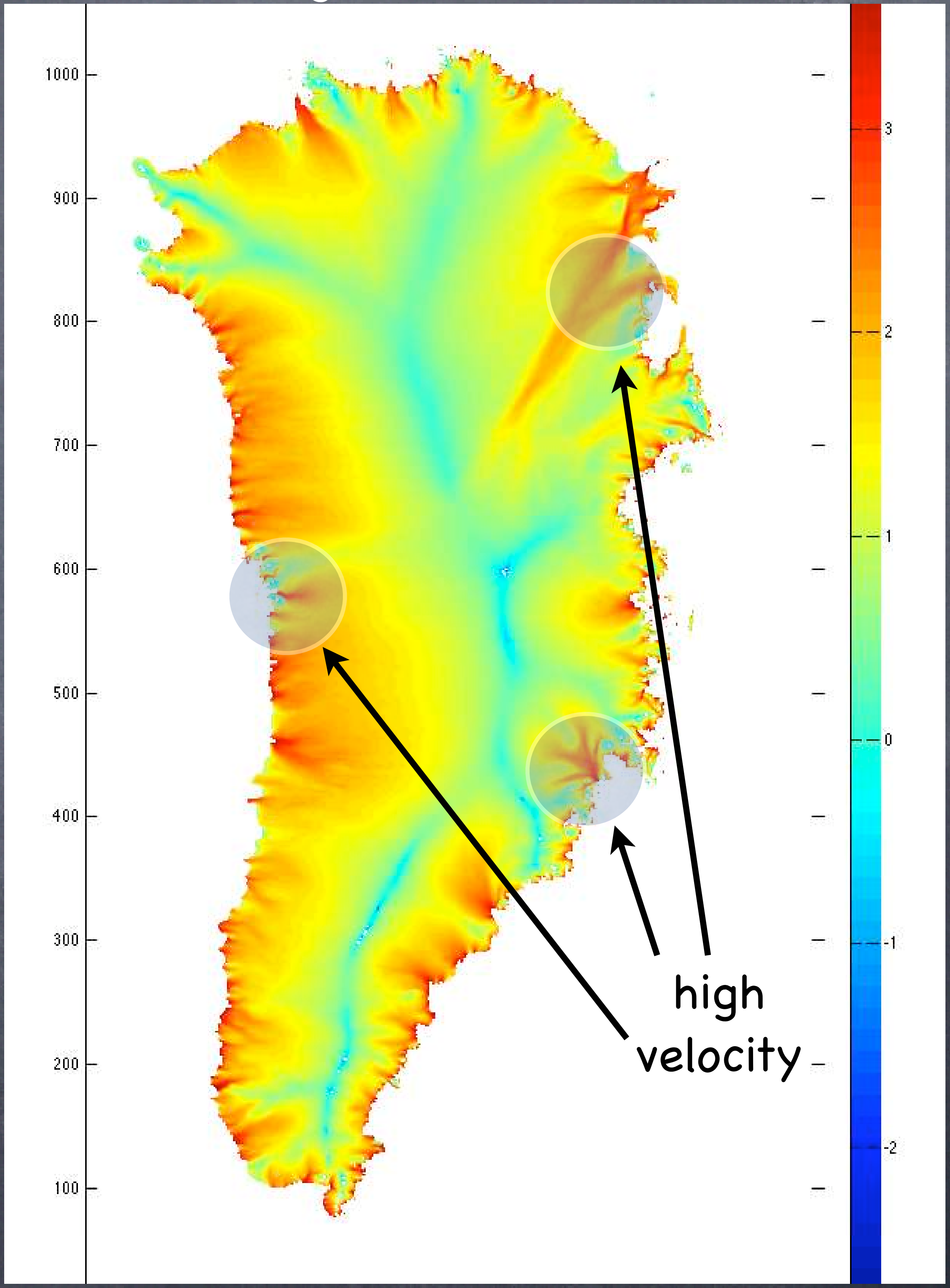
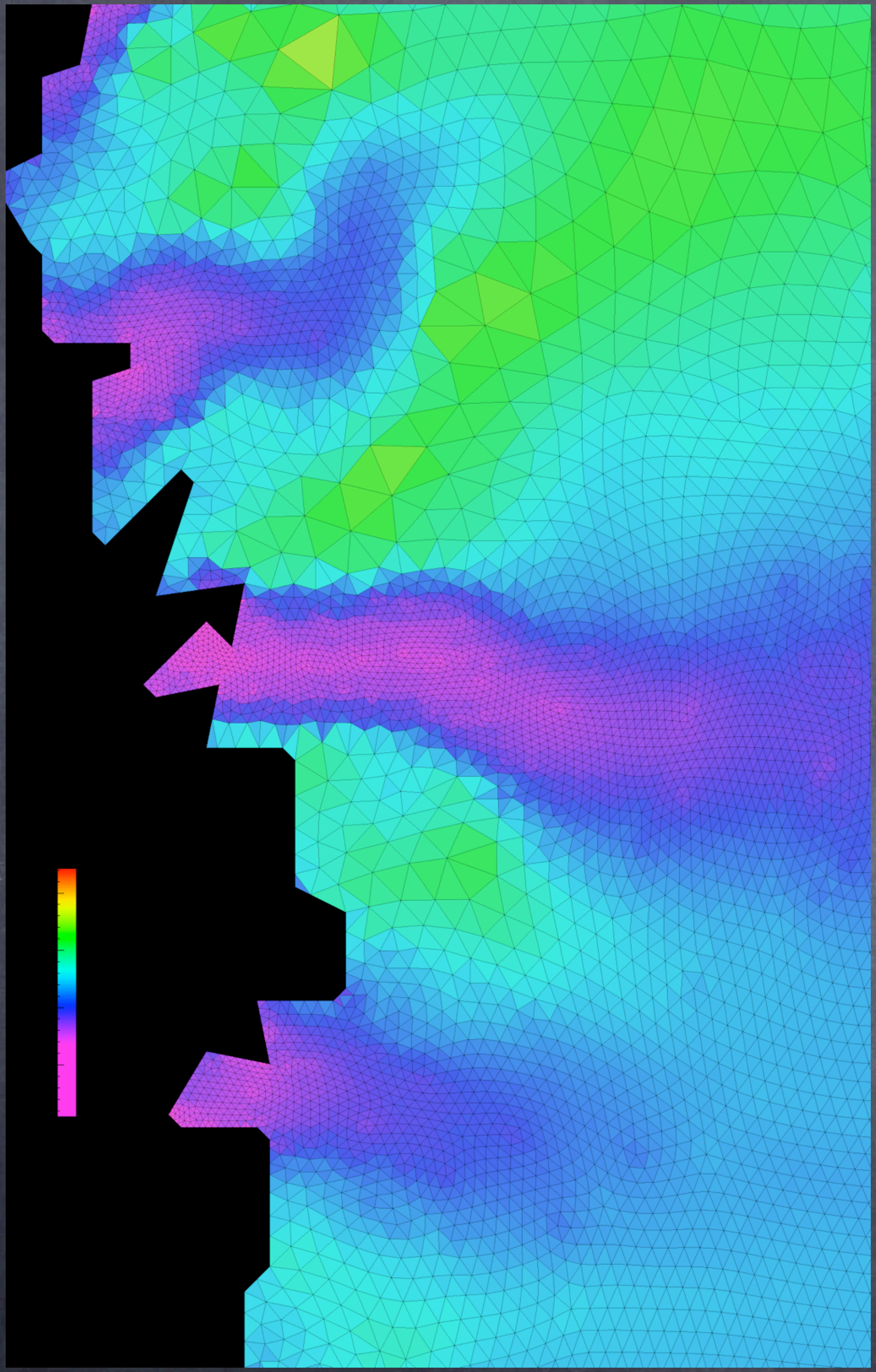
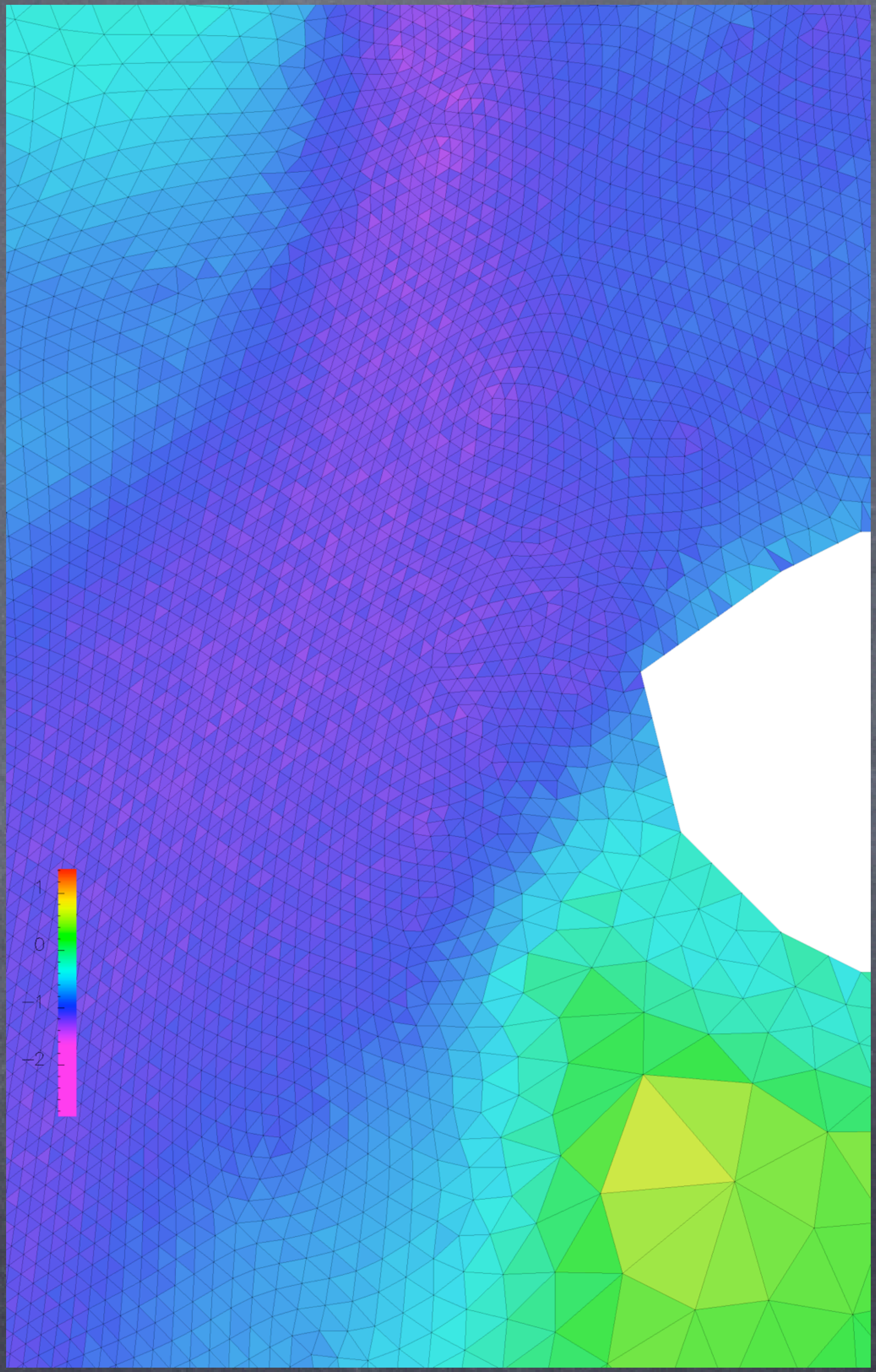
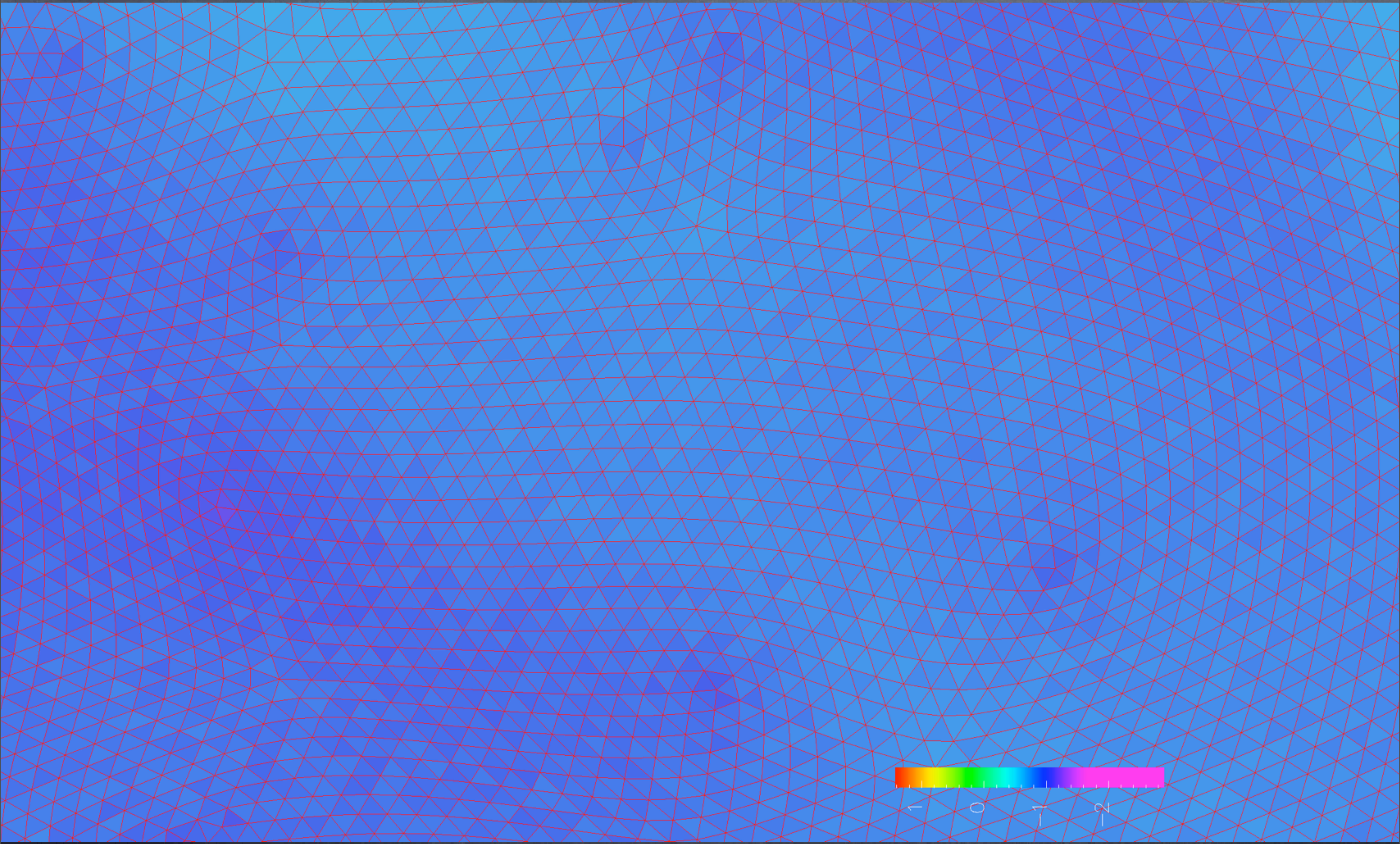


A look at the meshes ....

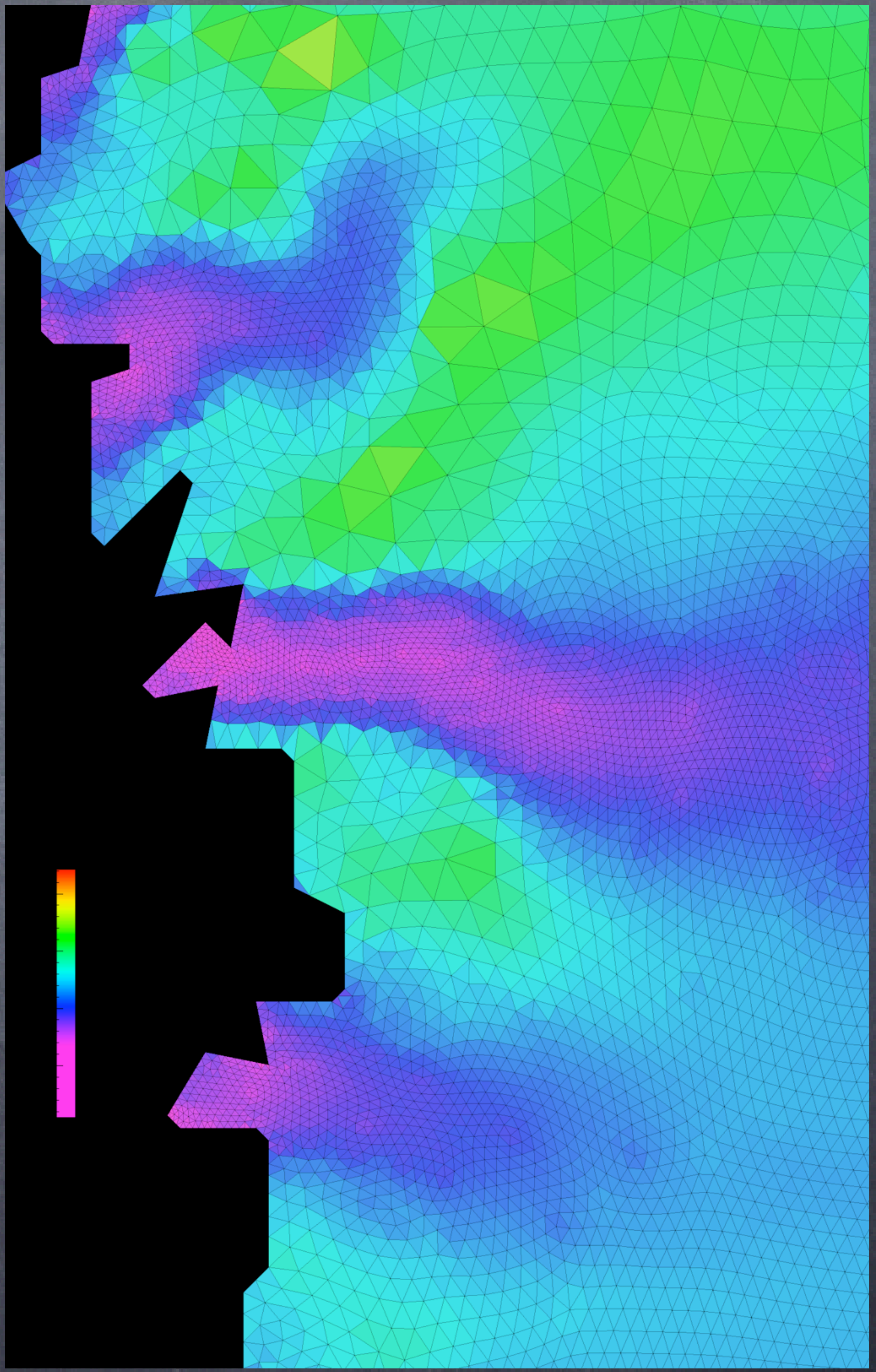




Globally, the mesh varies tremendously.
Locally, the mesh looks uniform.



Likely customer scenario ....



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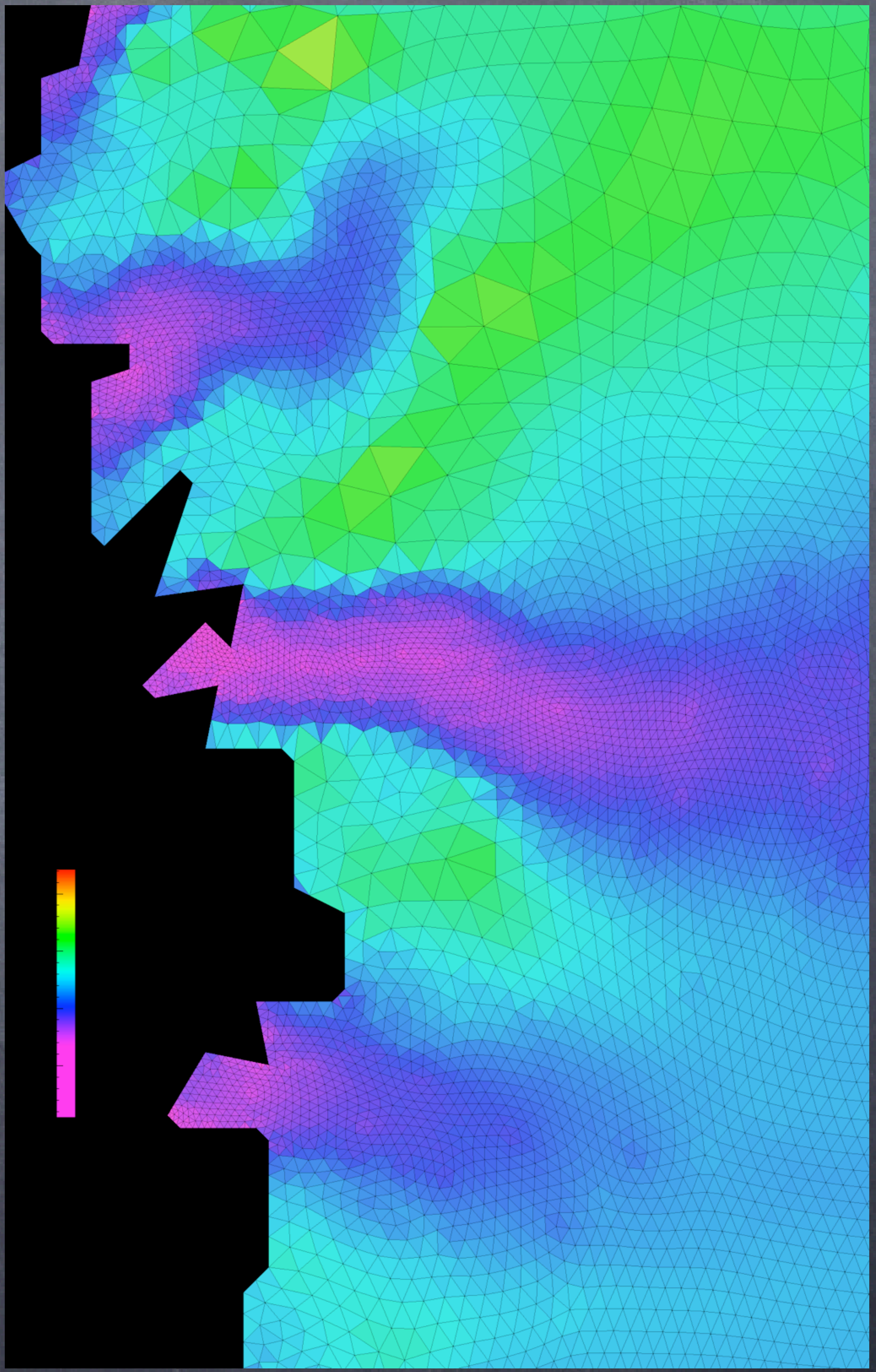

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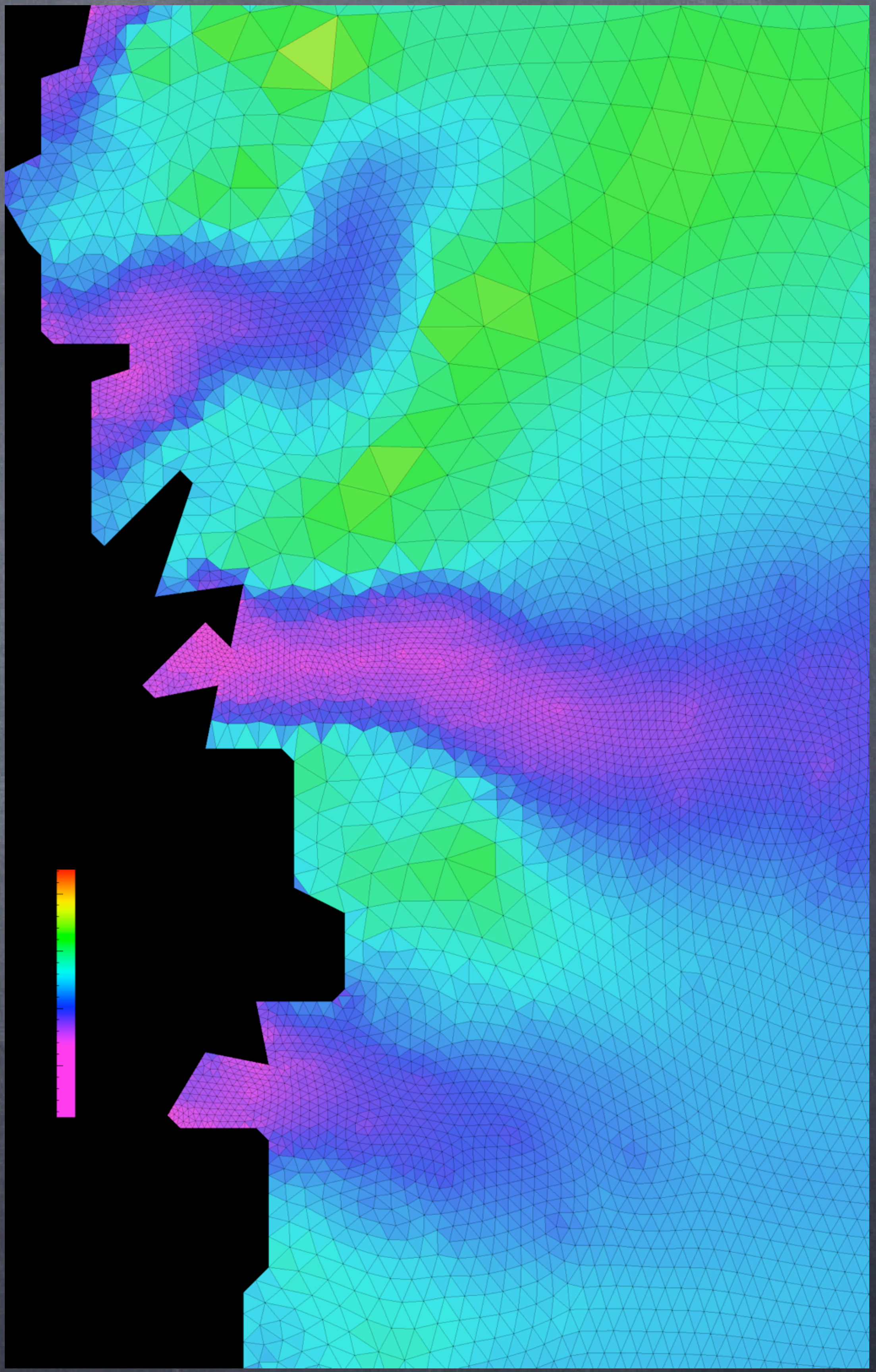

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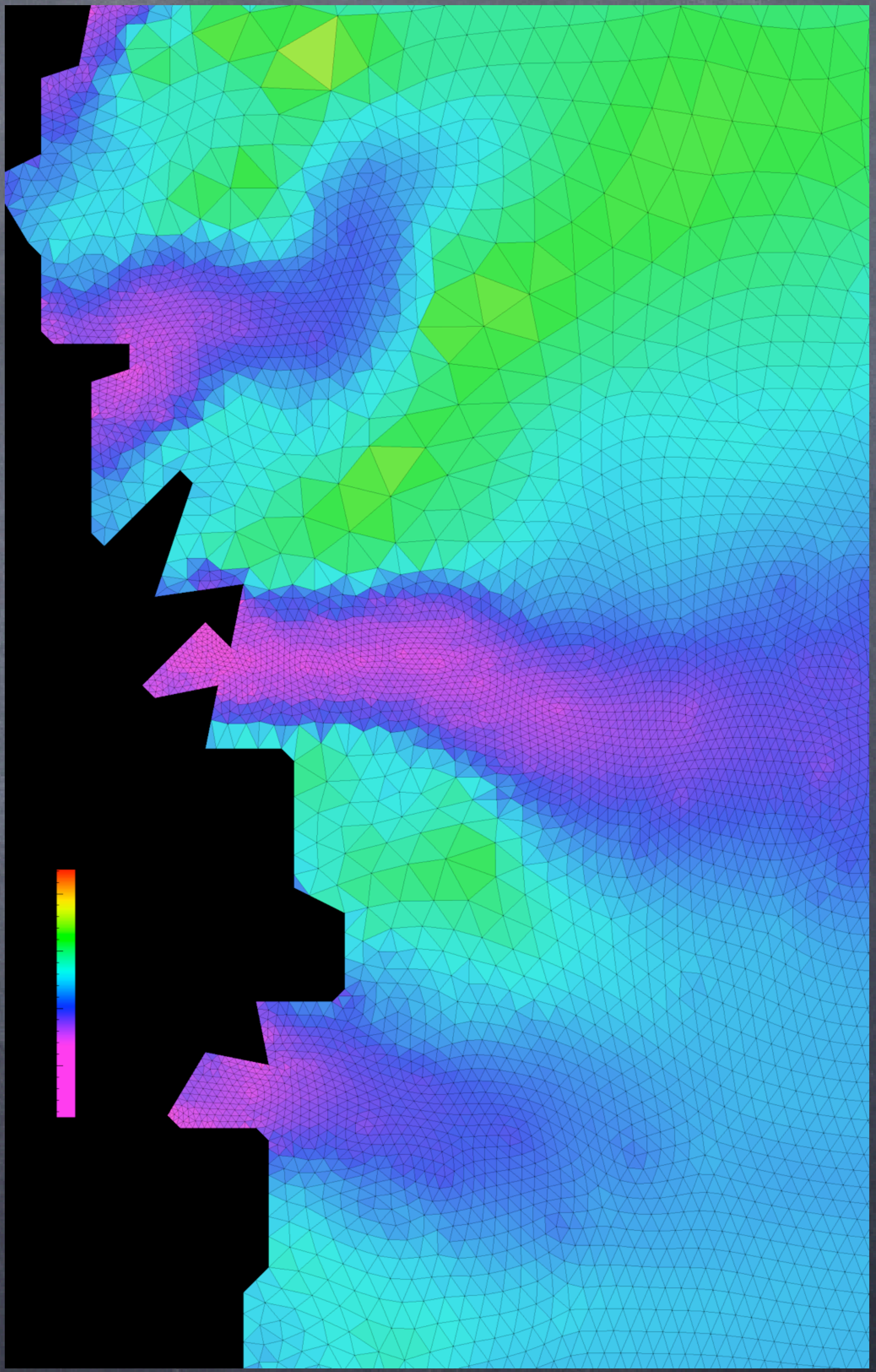
We can guarantee an even smoother grid by either adding more nodes or reducing the variation in our proxy density function.



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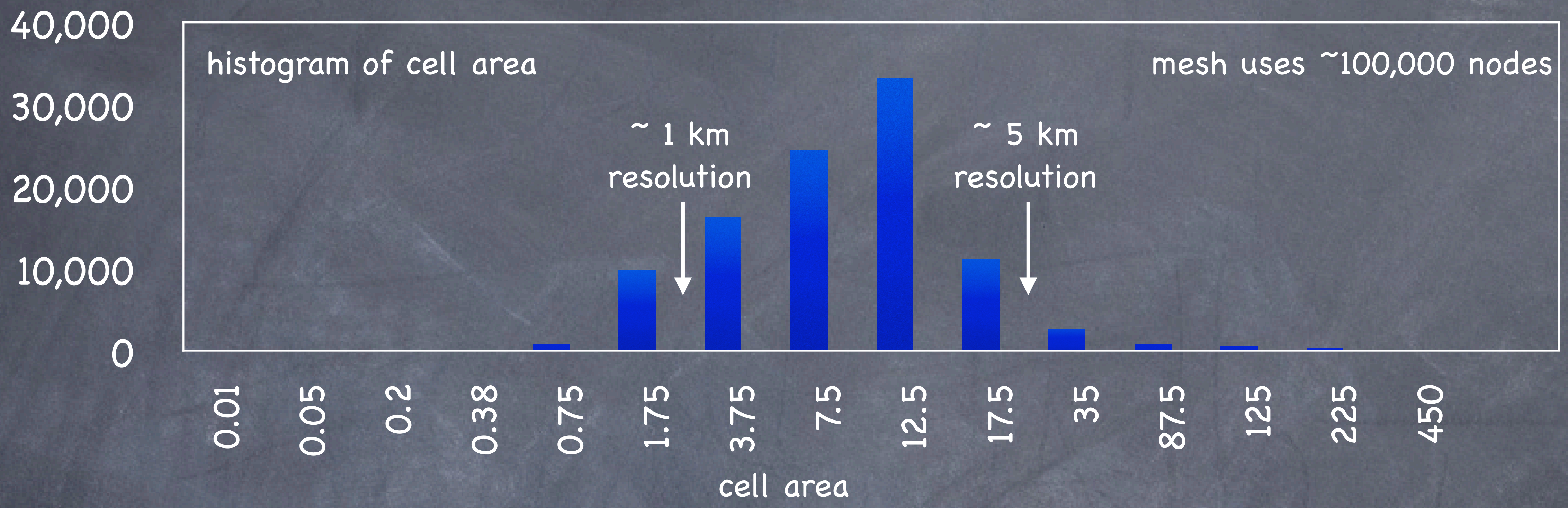

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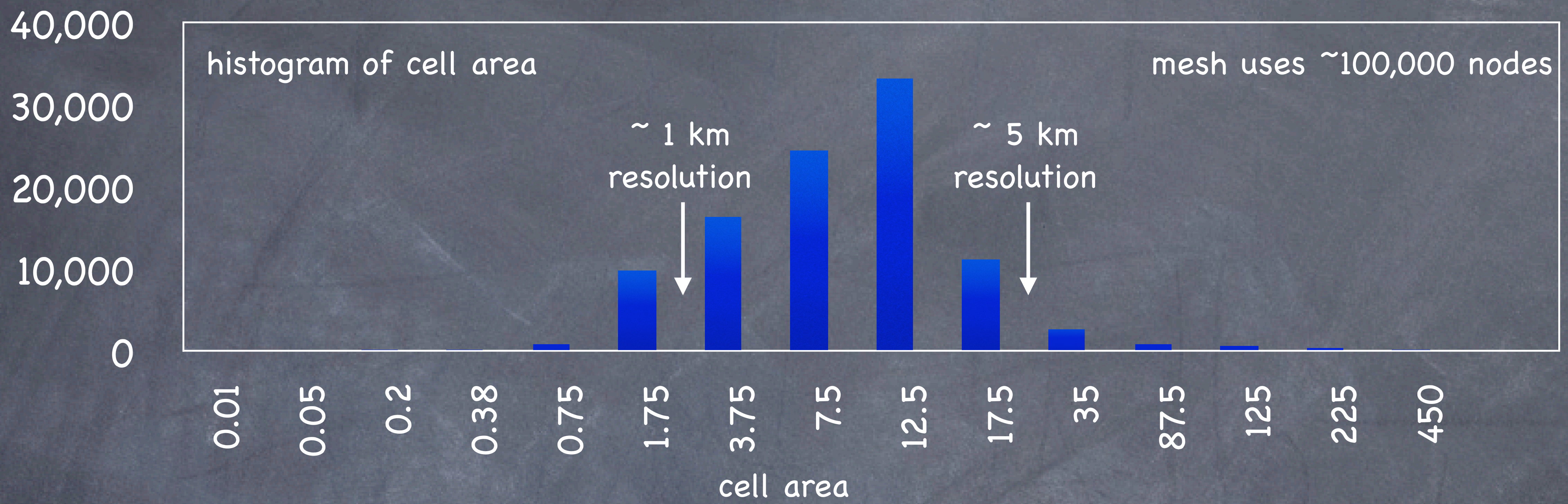

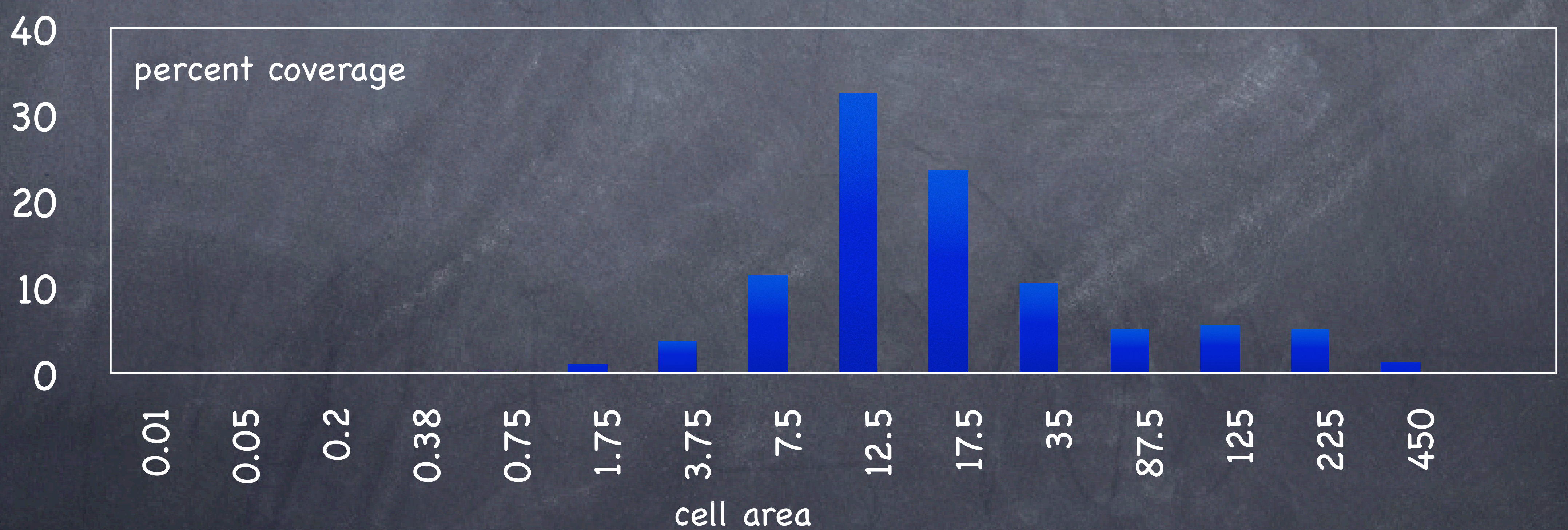

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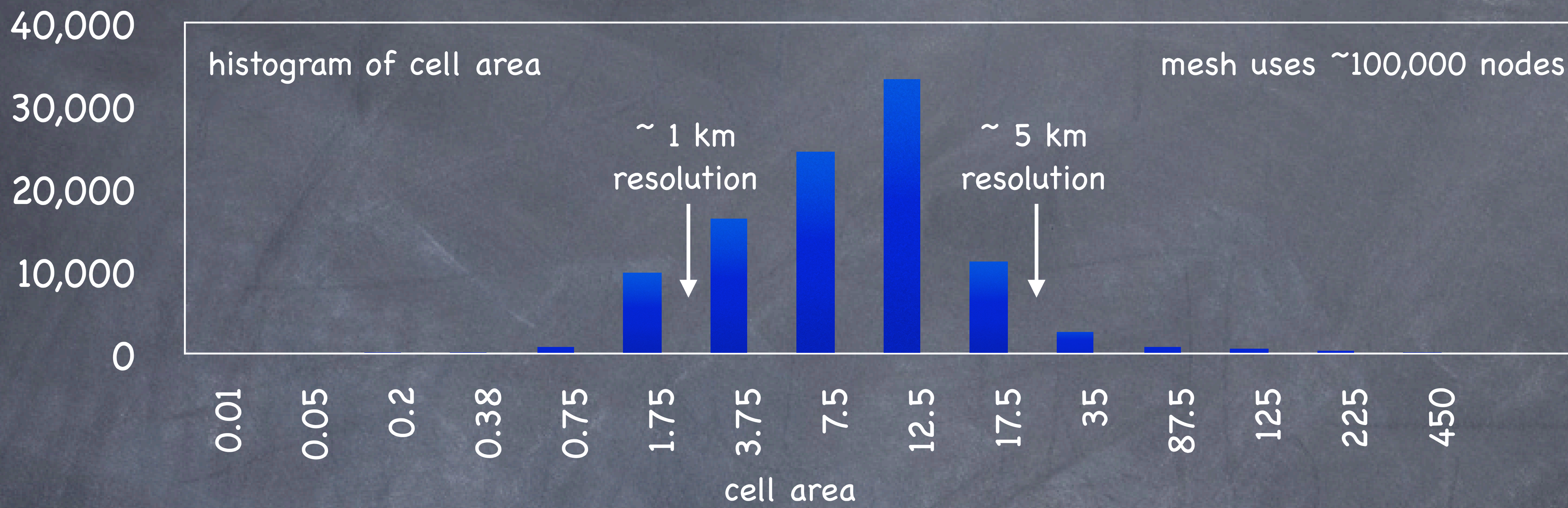

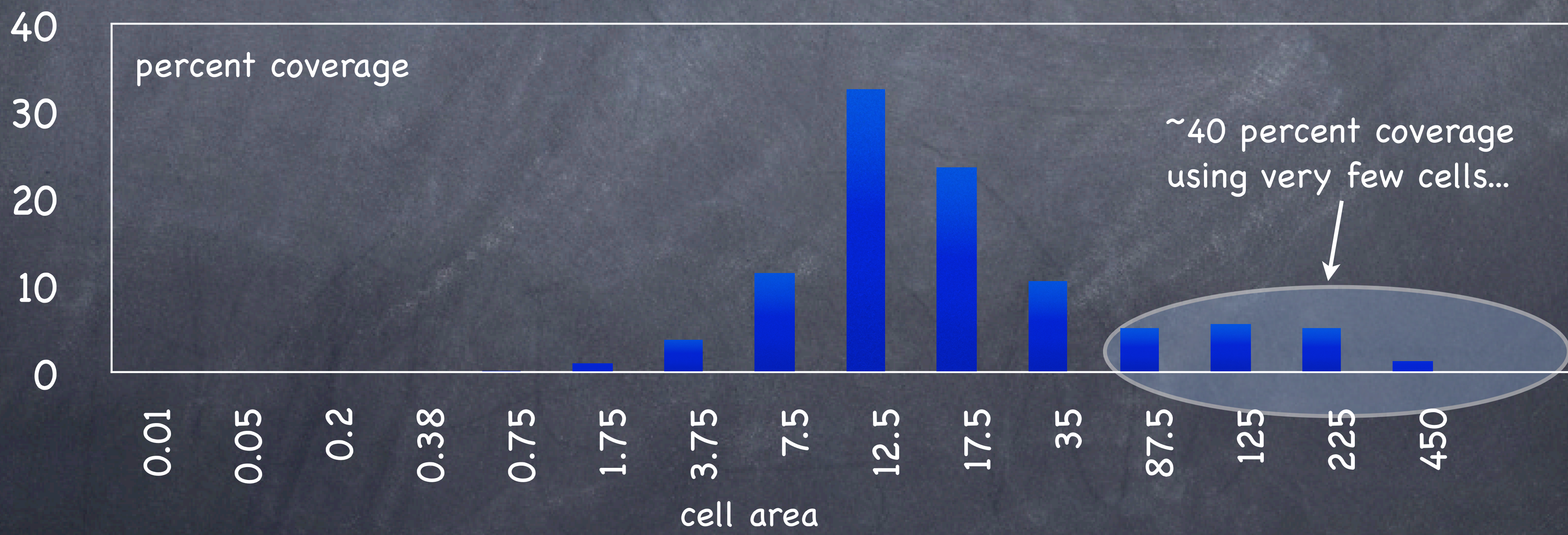

Where does “Mesh Generation” effort stand?

1) We have meshes to begin the discussion.

2) Our mesh generation is ahead of our model.

3) We are looking for a small set of users to collaborate with.

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1) Use the Stokes system because:
   a) It is our most valid representation of the dynamics.
   b) The stakes are too high to choose otherwise.
   c) We think we can do it in a computational tractable way.

In summary, our approach is the following:

1) Use the Stokes system because:
   a) It is our most valid representation of the dynamics.
   b) The stakes are too high to choose otherwise.
   c) We think we can do it in a computational tractable way.

2) Mitigate the cost of solving the Stokes system by:
   a) Developing a variable resolution mesh technology.
   b) Developing numerical methods that thrive on such meshes.

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Thanks!